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2002 J. Phys.: Condens. Matter 14 13899

(http://iopscience.iop.org/0953-8984/14/50/315)

INSTITUTE OF PHYSICS PUBLISHING JOURNAL OF PHYSICS: CONDENSED MATTER

J. Phys.: Condens. Matter **14** (2002) 13899–13904 PII: S0953-8984(02)40187-7

Generation of three primary colours through coupled quasi-phase-matched processes

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Received 2 August 2002
Published 6 December 2002
Online at stacks.iop.org/JPhysCM/14/13899

Abstract

Two coupled quasi-phase-matched processes, i.e., optical parametric generation and sum-frequency generation, can be used to obtain efficient generation of three primary colours in a single optical superlattice. In this paper, we solve the coupled equations obtained in the plane-wave approximation and point out the possibility of realizing equal outputs of red, green, and blue light.

Quasi-phase-matching theory [1] and the concept of the dielectric superlattice [2] have led to the devising of novel and efficient methods for frequency conversion. Through the electric field poling technique [3, 4], the domains of ferroelectric crystals (such as $LiNbO_3$ or $LiTaO_3$ (LT)) can be modulated artificially [5–9]. In general, a periodic optical superlattice (OSL) can provide one reciprocal vector to compensate the mismatch of wavevectors in the optical parametric process, while a quasi- or double-periodic [7–9] one can provide more, so coupled optical parametric processes may occur efficiently. In this way, enhanced third-harmonic generation (THG) [7, 9] and high-order-harmonic generation [10] can be obtained.

Recently, the generation of three primary colours (TPC) has attracted much attention because of its potential applications in future optoelectronic technology. Through optical frequency conversion, the efficient generation of red [11], green [7], and blue [12] light has been realized. Moreover, the simultaneous output of TPC has been demonstrated [13, 14]. Reference [14] presented a scheme with two periodic OSL used for optical parametric generation and sum-frequency generation. But the experimental result shows that the conversion efficiency of the blue light is much lower than that of the red light, which is not desired in the design of TPC lasers. In this paper, two quasi-phase-matched (QPM) processes, i.e., optical parametric generation and sum-frequency generation, are coupled in a single OSL to obtain equal outputs of TPC, where two mismatches of wavevectors are simultaneously compensated by two reciprocal vectors (the wavelength of the pump is 532 nm). We have solved the coupled equations obtained in the plane-wave approximation both numerically and analytically. The conditions for realizing equal outputs of TPC have been discussed. Finally, we give a simple example of our design.

Without loss of generality, we take LT as the working material. In order to use the largest nonlinear optical coefficient d_{33}, the pump beam is set to propagate in the x-direction with its polarization along the optical axis of the crystal LT. In the plane-wave approximation, the equations for two coupled QPM processes ($\omega_1 \to \omega_2 + \omega_3, \omega_1 + \omega_3 \to \omega_4$) are as follows:

$$
\begin{aligned}
\frac{\mathrm{d}A_1}{\mathrm{d}x} &= -\mathrm{i}\alpha A_2 A_3 \mathrm{e}^{\mathrm{i}\,\Delta k_1 x} - \mathrm{i}\beta A_3^* A_4 \mathrm{e}^{-\mathrm{i}\,\Delta k_2 x} \\
\frac{\mathrm{d}A_2}{\mathrm{d}x} &= -\mathrm{i}\alpha A_1 A_3^* \mathrm{e}^{-\mathrm{i}\,\Delta k_1 x} \\
\frac{\mathrm{d}A_3}{\mathrm{d}x} &= -\mathrm{i}\alpha A_1 A_2^* \mathrm{e}^{-\mathrm{i}\,\Delta k_1 x} - \mathrm{i}\beta A_1^* A_4 \mathrm{e}^{-\mathrm{i}\,\Delta k_2 x} \\
\frac{\mathrm{d}A_4}{\mathrm{d}x} &= -\mathrm{i}\beta A_1 A_3 \mathrm{e}^{\mathrm{i}\,\Delta k_2 x}
\end{aligned} \tag{1}
$$

with

$$
\begin{aligned}
&A_i = \sqrt{n_i/\omega_i}\,E_i, \qquad i = 1, 2, 3, 4 \\
&\alpha = \frac{f_a d_{33}}{c}\sqrt{\frac{\omega_1\omega_2\omega_3}{n_1 n_2 n_3}}, \qquad \beta = \frac{f_b d_{33}}{c}\sqrt{\frac{\omega_1\omega_3\omega_4}{n_1 n_3 n_4}}. \\
&\Delta k_1 = k_1 - k_2 - k_3 - G_a, \qquad \Delta k_2 = k_4 - k_1 - k_3 - G_b.
\end{aligned}
$$

E_i, ω_i, n_i, and k_i ($i = 1, 2, 3, 4$ refer to green, red, infrared, and blue light, respectively) are the electric fields, the angular frequency, the refractive indices, and the wavevectors, respectively; α, β are the coupling coefficients, f_a, f_b are the Fourier coefficients, and G_a, G_b are the reciprocal vectors; c is the speed of light in vacuum, d_{33} is the nonlinear optical coefficient, and $*$ denotes complex conjugation. Here, we should point out that the following discussion is based on the exact QPM conditions, i.e., $\Delta k_1 = 0$ and $\Delta k_2 = 0$. Under these conditions, the coupling between the two processes is greatly enhanced.

Numerical calculations of equations (1) are performed with the following boundary conditions: $A_1(0) = 1.0$, $A_3(0) = 0.001$, and $A_2(0) = A_4(0) = 0$. In the calculations we find that the coupling processes always oscillate periodically, which is different from THG [15]. We also find that the ratio of two coupling coefficients $t \equiv \alpha/\beta$ plays a significant role in the four-wave interactions. Only when the ratio t is larger than unity can efficient generation of TPC be effected. When t equals a critical value $\tau \equiv \sqrt{\lambda_2/\lambda_4}$, the intensities of red and blue light are equal throughout the crystal. In the case for $1 < t < \tau$, the blue light is stronger than the red one, whereas for $t > \tau$, the red light is stronger. The results are shown in figure 1 for (a) $1 < t < \tau$, (b) $t = \tau$, (c) $t > \tau$. They are not difficult to understand. Inspecting equations (1) reveals that the right-hand side of the third equality is composed of two terms; the first term represents the parametric gain, the second the loss from the sum-frequency generation. If t equals unity, the gain is just balanced by the loss [16]. If t is less than unity, the loss exceeds the gain and the coupling processes will never be able to start. Therefore, the ratio t should be larger than unity to ensure that signal field is amplified. Furthermore, equations (1) give $|A_2| = t|A_4|$. Now, with $I_i(x) = \varepsilon_0 c\omega_i |A_i|^2/2$, one can see that the intensity ratio of the red and blue light relates only to the coupling coefficient ratio. Obviously, when $t = \tau$, the intensities of the two types of light are equal.

As can be seen from figure 1(b), equal outputs of TPC can be produced if the crystal length equals a critical length X_O. And X_O is determined by solving equations (1). For convenience, the A_j are rewritten in the form $A_j = y_j \exp(\mathrm{i}\varphi_j)$ (then the y_j are all real; $j = 1, 2, 3, 4$) and substituted into equations (1). With the boundary conditions $y_1(0) = y_{10}$, $y_3(0) = y_{30}$

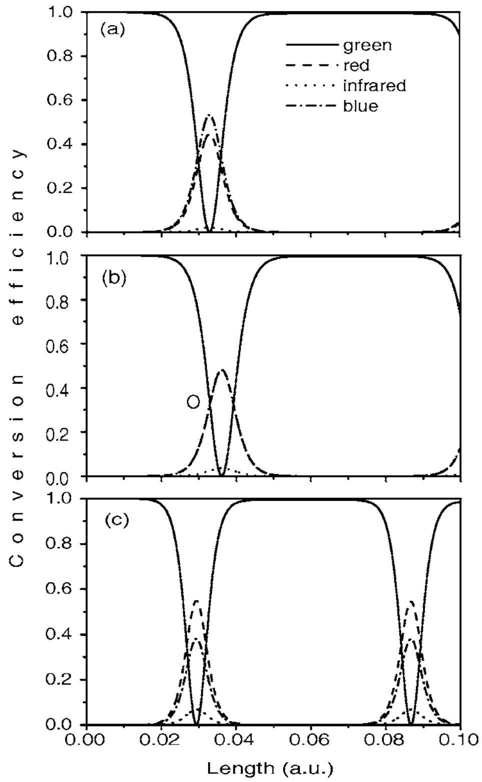

Figure 1. The dependence of the conversion efficiency on the crystal length: (a) $1 < t < \tau$, the blue light is stronger than the red; (b) $t = \tau$, the conversion efficiency of red and blue lights are equal and the two curves coincide; (c) $t > \tau$, the red light is stronger.

with y_{30} much smaller than y_{10} (since in the experiment y_{30} is usually produced from noise, this condition can always be satisfied), and $y_2(0) = y_4(0) = 0$, the approximated solution of equations (1) can be expressed as

$$y_1(x) \approx \begin{cases} \dfrac{4y_{10}^2 - a\mathrm{e}^{2kx}}{4y_{10}^2 + a\mathrm{e}^{2kx}}\sqrt{y_{10}^2 + a} & 0 \leqslant x \leqslant T/2 \\ -\dfrac{4y_{10}^2 - a\mathrm{e}^{2k(x-T/2)}}{4y_{10}^2 + a\mathrm{e}^{2k(x-T/2)}}\sqrt{y_{10}^2 + a} & T/2 \leqslant x \leqslant T \end{cases}$$

$$y_2^2(x) = \frac{t^2}{t^2+1}[y_{10}^2 - y_1^2(x)] \tag{2}$$

$$y_3^2(x) = \frac{t^2-1}{t^2+1}[y_{10}^2 - y_1^2(x)] + y_{30}^2$$

$$y_4^2(x) = \frac{1}{t^2+1}[y_{10}^2 - y_1^2(x)]$$

with

$$a = \frac{t^2+1}{t^2-1}\frac{y_{30}^2}{2}, \qquad k = \sqrt{\alpha^2 - \beta^2}\,y_{10}, \qquad T = \frac{2}{k}\ln\left(\frac{4}{a}y_{10}^2\right).$$

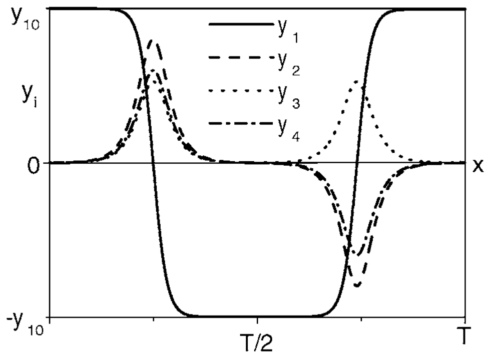

Figure 2. The functional relation between y_i and the crystal length. y_i oscillates periodically with a period T.

In the derivation, some terms involving y_{30}^2 have been omitted. This is reasonable, since y_{30} is very small. The solution indicates that the coupling processes oscillate periodically with a period T. Figure 2 shows qualitatively the functional relation between y_i and the crystal length, which is in accordance with figure 1. In a period, $y_1(x)$ oscillates between y_{10} and $-y_{10}$. From the viewpoint of energy conversion, a small signal y_{30} produced from noise may result in depletion of the pump wave; simultaneously the other waves are amplified with the interactions. When $y_1(x) = 0$, y_2, y_3, and y_4 reach their maxima, with the pump's energy being converted completely to the other waves. And when $y_1(x) = \pm y_{10}$, all the energy is transferred back to the pump again. It is evident that $y_1(x)$ oscillates once in a period with energy oscillating twice. Additionally, it is worth noting that the above solution (2) is still valid for a single optical parametric generation as long as we set $\beta \to 0$, $t \to \infty$.

Setting $I_1(x) = I_2(x)$ and $t = \tau$, and using equations (2), the critical length X_O can be determined as

$$X_O = \frac{1}{2k} \ln \frac{4y_{10}^2(r-1)}{a(r+1)} \tag{3}$$

with

$$r = \sqrt{2 + \frac{\tau^2 + \tau^{-2}}{2}}.$$

We find that the critical length X_O is related to the parameters α, β and y_{10}, y_{30}. The larger y_{10}, the smaller X_O (see figure 3). Thus, in general cases, we can adjust y_{10} to make X_O equal to the crystal length. Furthermore, y_{10} can be calculated from

$$y_{10}^2 = \frac{2}{\varepsilon_0 c \omega_1} \frac{\bar{P}}{swv}. \tag{4}$$

$\bar{P}$ is the average power, s is the area of the pump beam, v is the repetition rate, and w is the duration of the pulse. Then, the parameters $\bar{P}$, s, w, v will be adjustable to obtain an optimum y_{10}. With the crystal length fixed, the dependence of the conversion efficiency on y_{10} is shown in figure 4. The conversion efficiency of red, blue, and green light will oscillate with y_{10} and the same conversion efficiency can be obtained if y_{10} is suitable.

Here an example of our design is given. To simplify the design process, a periodic OSL is used to generate equal power outputs of TPC, which provides second- and third-order reciprocal vectors to compensate the two wavevector mismatches simultaneously. In the calculations, the Sellmeier equation for LT is used [17]. The wavelengths of green, red, infrared, and

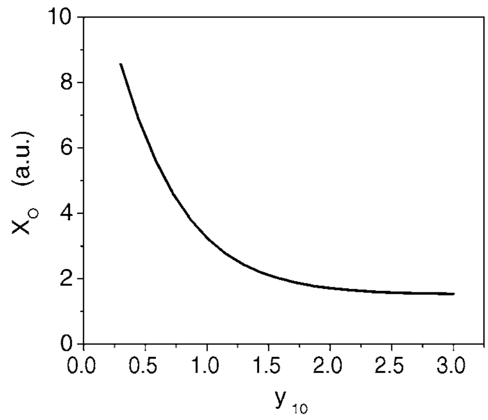

Figure 3. The critical length X_O versus y_{10}. The larger y_{10}, the smaller X_O is.

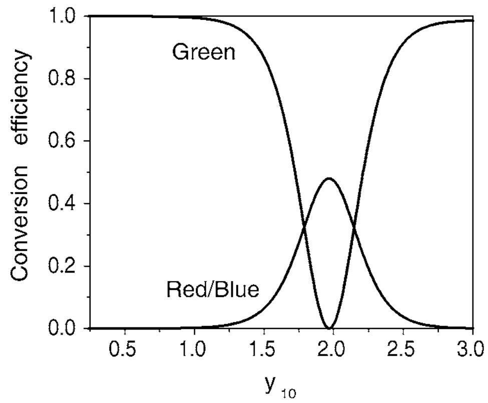

Figure 4. The dependence of the conversion efficiency on y_{10} $(t = \tau)$. An optimum y_{10} is needed for equal outputs of TPC if the crystal length is fixed.

blue light are chosen as 532, 658, 2778.2, and 446.5 nm, respectively. The temperature is set at $T = 93.5\,^{\circ}\mathrm{C}$. Under these conditions, the period of the OSL is determined as 21.597 μm, the duty cycle $D = 0.193$, and the Fourier coefficients $f_2 = 0.298$, $f_3 = 0.206$ with $f_m = (2/\pi m)\sin(mD\pi)$. As can be seen from equation (3) the critical length X_O of the OSL is related to y_{10} and y_{30}. We assume that the 532 nm pump wave comes from the second-harmonic generation of a ps Nd:YAG laser with a pulse width of 43 ps and a repetition rate of 10 Hz. The radius of the beam waist inside the OSL is 0.1 mm and the average power of the pump is 2 mW [14]. Then y_{10} can be calculated from equation (4) to be 5.61. And y_{30} is determined by the noise intensity $I_{30} = \varepsilon_0 c\omega_3 y_{30}^2/2 = N_3\hbar\omega_3$. Suppose that at the beginning of the optical parametric generation there is only one noise photon, i.e., $N_3 = 1$. Then y_{30} is 2.82×10^{-16}. According to equation (3), the critical length is $X_O = 3.25$ cm.

In summary, the generation of TPC with coupled optical parametric generation and sum-frequency generation has been discussed theoretically. In the plane-wave approximation, we have solved the coupled equations both numerically and analytically. The calculations show that the generation of TPC is greatly influenced by the coupling coefficients and the crystal length. In principle, equal outputs of TPC can be achieved if both the coupling coefficient ratio and the crystal length are chosen appropriately. The results identify the possibility of realizing an ideal generation of TPC with a special design of the microstructures.

Acknowledgments

This work was supported by a grant for the State Key Programme for Basic Research of China and by the National Natural Science Foundation of China (No 10021001).

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